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# The nonlinear carrier transport in a bipolar semiconductor sample

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## Abstract

A theory of formation of the voltage across a bipolar semiconductor sample due to the current flow accounting for the energy band bending near the semiconductor surfaces is presented. The non-equilibrium space charge layers near the sample surfaces and the boundary conditions in the real metal–semiconductor junction have been taken into account. It is shown that the voltage–current relation of a thin sample at weak injection differs essentially from the classical Ohm’s law and becomes nonlinear for certain semiconductor surface parameters. Complex voltage–current relations and the photo-induced electromotive force measurements allow determining the surface recombination rate in the real metal–semiconductor junction and the semiconductor surface potential.

## 1. Introduction

Charge-carrier transport lies as the basis of the electrical behavior of any semiconductor device. Despite efforts to correctly model transport in semiconductors over the years, many questions remain open in the study of transport phenomena. One of the questions is the electric current flow through the metal–semiconductor junction (MSJ). The linear carrier transport in a bipolar semiconductor sample, through which a weak current flows, was studied in [1]. The model of [1] assumes local electroneutrality in the bulk of a semiconductor and is correct in the case of flat energy bands. However, energy band bending usually occurs near the MSJ [2]. It follows from the results of [3–7] that the local electroneutrality approximation is insufficient for the study of the problems related to the energy band bending influence on the non-equilibrium carrier density distribution. The current flow through a bipolar semiconductor sample is one of these problems. It is shown in [3–5] that the non-equilibrium space charge layer (SCL) arises at a distance of several Debye lengths [3] from the real MSJ. This non-equilibrium charge creates the so-called barrier electromotive force (emf) [8], which can significantly change as well as the Dember emf [3, 4], the thermo emf [6] and the Hall emf [5] values. All these emf values have been calculated assuming a small change of carrier density from equilibrium in the volume of the sample and in the SCL. This condition essentially limits the light intensity, the Lorenz force and the temperature gradient when the surface potential [2] is negative. Note that the usually used condition of weak photo-excitation assumes a small variation of the electron density only in the volume

of the sample. In this case the system of continuity equations becomes nonlinear [7] with respect to the SCL non-equilibrium carrier density. Therefore, the voltage–current relation of the sample can be nonlinear in the case of weak injection for certain values of the semiconductor surface potential.

The aim of this paper is to study the role of the non-equilibrium charge in the formation of the nonlinear voltage–current relation of a bipolar semiconductor sample.

## 2. Theory

Let us consider a bipolar semiconductor plate  $-a \leq x \leq a$ . Metallic contacts are placed on the surfaces  $x = \pm a$  of the sample. The electric current flows through the semiconductor. We suppose that carrier injection is low, i.e. the non-equilibrium carrier density is significantly less than the equilibrium density in the bulk of the sample. The metallic contacts of the sample have an ideal thermal contact with the thermostat, whose temperature is  $T$ .

The non-equilibrium densities of electrons,  $\delta n$ , and holes,  $\delta p$ , as well as the non-equilibrium electric potential,  $\delta\varphi$ , are obtained from solution of the continuity equations [3] and the Poisson equation

$$\frac{1}{e} \frac{dj_n}{dx} - \frac{\delta n}{\tau_n} - \frac{\delta p}{\tau_p} = 0, \quad (1)$$

$$\frac{1}{e} \frac{dj_p}{dx} + \frac{\delta n}{\tau_n} + \frac{\delta p}{\tau_p} = 0, \quad (2)$$

$$\frac{d^2\delta\varphi}{dx^2} = \frac{e}{\epsilon\epsilon_0}(\delta n - \delta p), \quad (3)$$

where  $-e$  is the electron charge,  $j_n, j_p$  are the electron and hole current densities,  $\tau_n(\tau_p)$  is the parameter characterizing the electron (hole) bulk recombination rate,  $\varepsilon$  is the semiconductor electrical permittivity and  $\varepsilon_0$  is the vacuum permittivity. The analytical expression of the recombination rate is obtained on the basis of irreversible thermodynamics [9] and on the basis of the Shockley–Read model [10].

In our case considered here the expressions for the  $x$  component of partial currents take the form [2]

$$\begin{aligned} j_n &= -e\mu_n n \frac{d\varphi}{dx} + \mu_n kT \frac{dn}{dx} \\ j_p &= -e\mu_p p \frac{d\varphi}{dx} - \mu_p kT \frac{dp}{dx}, \end{aligned} \quad (4)$$

where  $\mu_n(\mu_p)$  is the electron (hole) mobility,  $n(x), p(x)$  are the density of electrons and holes accordingly,  $\varphi$  is the electric potential,  $k$  is the Boltzmann constant and  $T$  is the temperature of the semiconductor.

The boundary conditions (BCs) in the real MSJ are obtained in [3, 11]

$$j_n(\pm a) = j_0 \mp ev_p \delta p(\pm a), \quad (5)$$

$$\delta n(\pm a) = 0, \quad (6)$$

$$\delta\varphi_M(\pm a) = \delta\varphi(\pm a) \mp j_0/\sigma_n^S, \quad (7)$$

where  $v_p$  is the surface recombination rate (SRR),  $j_0$  is the current density,  $\sigma_n^S$  is the electron surface conductivity [3, 11] and  $\delta\varphi_M$  is the change of electric potential of metallic contact caused by the current  $j_0$ . For simplicity we assume that the SRR values at the surfaces  $x = \pm a$  coincide.

The BCs can be explained as follows: the non-equilibrium electrons can cross the MSJ (the electron surface conductivity [3] is large enough) and therefore do not accumulate on the surfaces  $x = \pm a$ . The non-equilibrium holes accumulate and recombine on the surfaces  $x = \pm a$  because they do not cross the MSJ (there are no holes in the metal). Note that the MSJ thickness is significantly less than the Debye length in the model considered [11]. In the local electroneutrality model the MSJ thickness is much more than the Debye length because the BCs have been formulated at a virtual surface which is formed at a distance of several Debye lengths from the real MSJ [2, 8]. Two important conclusions follow from this difference. Firstly, the parameter  $v_p$  characterizes the SRR in the real MSJ. Secondly, the electron surface resistance [3, 11], which causes the electric potential step across the MSJ, is small enough. Let us evaluate the electron surface resistance of the unit surface square, taking into account that the thickness of the real MSJ  $\Delta$  is significantly less than the Debye length  $r_D$ :

$$\begin{aligned} (\sigma_n^S)^{-1} &= \lim_{\xi \rightarrow 0} \int_{-a-\xi}^{-a+\xi} \sigma_{n,\text{eq}}^{-1} dx \\ &\approx \frac{\Delta}{\sqrt{\sigma_M \sigma_{n,\text{eq}}(0)}} \ll r_D \sigma_n^{-1} \ll a \sigma_n^{-1}, \end{aligned}$$

where  $\sigma_{n,\text{eq}}$  is the equilibrium electron conductivity in the MSJ,  $\sigma_M$  is the metal conductivity and  $\sigma_n$  is the equilibrium electron conductivity in the bulk of the semiconductor. Thus

the electron surface resistance is negligible compared with the bulk one. Therefore, we neglect below the electric potential step across the MSJ. Note that in the local electroneutrality model the surface resistance may be comparable with the bulk one.

In most semiconductors the diffusion length  $\lambda$  significantly exceeds the Debye length  $r_D$ . Under this condition the solution of equations (1)–(4) could be obtained as a sum of three modes: the classical bulk mode, the diffusion–recombination (DR) mode and the screening (S) mode. These modes are denoted by subscripts C, R and S accordingly:

$$\begin{aligned} \delta n &= \delta n_R + \delta n_S, \\ \delta p &= \delta p_R + \delta p_S, \end{aligned} \quad (8)$$

$$\delta\varphi = \varphi_C + \delta\varphi_R + \delta\varphi_S.$$

The C and DR modes are obtained from the solution of equations (1), (2) and (4), taking into account that  $\lambda$  is the characteristic DR mode decay length, the classic bulk mode is linear in the coordinate  $x$ , and the inequality  $\lambda \gg r_D$  is valid. Therefore, deducing these modes we can assume that  $n = n_0 + \delta n_R$ ,  $p = p_0 + \delta p_R$  and  $\varphi = \varphi_C + \delta\varphi_R$ , where  $n_0$  ( $p_0$ ) is the equilibrium density of electrons (holes) in the bulk of the sample. It follows from equations (1) and (2) that

$$j_n + j_p = j_0. \quad (9)$$

On account of the physical symmetry of the problem we can suppose the electric potential is equal to zero on the plane  $x = 0$ . Therefore we derive from equations (4) and (9)

$$\varphi_C = -j_0 \sigma^{-1} x, \quad (10)$$

$$\delta\varphi_R = (\mu_n - \mu_p) kT \sigma^{-1} \delta n_R, \quad (11)$$

$$j_n = \frac{\sigma_n}{\sigma} j_0 + eD \frac{d\delta n_R}{dx}, \quad (12)$$

where  $\sigma_n = en_0\mu_n$  is the equilibrium electron conductivity,  $\sigma_p = ep_0\mu_p$  is the equilibrium hole conductivity,  $\sigma = \sigma_n + \sigma_p$  and  $D = (kT/e\sigma)(\sigma_n\mu_p + \sigma_p\mu_n)$  is the bipolar diffusion coefficient.

The continuity equation for the DR mode one obtains from equations (1), (2) and (12):

$$\frac{d^2\delta n_R}{dx^2} - \frac{\delta n_R}{\lambda^2} = 0, \quad (13)$$

where  $\lambda = \sqrt{D\tau}$  is the diffusion length and  $\tau = \tau_n\tau_p/(\tau_n + \tau_p)$  is the lifetime of the electron–hole pairs in the bulk of the sample. The non-equilibrium hole density  $\delta p_R$  satisfies the same equation.

On account of the BC (5) symmetry the solution of equation (17) takes the form

$$\delta n_R = \delta n_R(a) \frac{\sinh(x/\lambda)}{\sinh(a/\lambda)}, \quad (14)$$

$$\delta p_R = (1 - \gamma)\delta n_R,$$

where  $\gamma = (\mu_n - \mu_p) \frac{\varepsilon\varepsilon_0 kT}{\lambda^2 e\sigma} \propto \frac{r_D^2}{\lambda^2} \ll 1$ .

The characteristic S mode decay length is the Debye length  $r_D$  and the characteristic DR mode decay length is the diffusion length  $\lambda$ . On account of the inequality  $r_D \ll \lambda$  we can neglect the bulk recombination deriving the S mode. Therefore the continuity equations (1) and (2) for the S mode take the form

$$\frac{dj_{nS}}{dx} = 0, \quad \frac{dj_{pS}}{dx} = 0. \quad (15)$$

The solution of equation (15) is

$$j_{nS} = \text{const} = 0, \quad j_{pS} = \text{const} = 0, \quad (16)$$

because the S mode is not equal to zero in a layer of several  $r_D$  distant from the surfaces  $x = \pm a$ . Let us obtain the S mode near the surface  $x = a$ , denoted as  $\delta n_S^+$ ,  $\delta p_S^+$  and  $\delta \varphi_S^+$ . The current flowing through the sample is assumed positive ( $j_0 > 0$ ). It follows from equations (5), (12) and (14) at negligible SRR ( $v_p = 0$ ) that  $\delta n_R(a) = \delta n_R^+ > 0$ . Owing to the inequality  $r_D \ll \lambda$  the DR mode does not change in the S mode existence region. Therefore we can consider the densities  $\tilde{n}_+ = n_{\text{eq}} + \delta n_R^+$  and  $\tilde{p}_+ = p_{\text{eq}} + \delta n_R^+$  as the equilibrium electron and hole densities of the S mode near the surface  $x = a$ . In other words the electron and hole densities of the S mode  $\delta n_S^+$  and  $\delta p_S^+$  are counted off from the levels  $\tilde{n}_+$  and  $\tilde{p}_+$  accordingly. Here  $n_{\text{eq}}(p_{\text{eq}})$  is the real equilibrium density of electrons (holes).

Solving equations (4) and (16) we obtain the S mode near the surface  $x = a$ :

$$\begin{aligned} \delta n_S^+ &= (n_{\text{eq}} + \delta n_R^+) \left[ \exp\left(\frac{e\delta\varphi_S^+}{kT}\right) - 1 \right], \\ \delta p_S^+ &= (p_{\text{eq}} + \delta n_R^+) \left[ \exp\left(-\frac{e\delta\varphi_S^+}{kT}\right) - 1 \right]. \end{aligned} \quad (17)$$

One deduces from equations (17) and (6):

$$\delta p_S^+(a) = [p_{\text{eq}}^+ + \delta n_R^+] \frac{\delta n_R^+}{n_{\text{eq}}^+}, \quad (18)$$

$$\delta \varphi_S^+(a) = -\frac{kT}{e} \ln \left[ 1 + \frac{\delta n_R^+}{n_{\text{eq}}^+} \right], \quad (19)$$

where  $n_{\text{eq}}^+ = n_0 \exp(e\varphi^S/kT)$ ,  $p_{\text{eq}}^+ = p_0 \exp(-e\varphi^S/kT)$  and  $\varphi^S$  is the surface potential (SP) [2].

The S mode near the surface  $x = -a$  (denoted as  $\delta n_S^-$ ,  $\delta p_S^-$  and  $\delta \varphi_S^-$ ) is obtained in the same way as the electron and hole densities of the S mode are counted off from the levels  $\tilde{n}_- = -(n_{\text{eq}} + \delta n_R^+)$  and  $\tilde{p}_- = -(p_{\text{eq}} + \delta n_R^+)$  accordingly. We derive from equations (4) and (16)

$$\begin{aligned} \delta n_S^- &= -\delta n_S^+, \\ \delta p_S^- &= -\delta p_S^+, \\ \delta \varphi_S^- &= -\delta \varphi_S^+. \end{aligned} \quad (20)$$

It follows from equation (19) that the S mode potential  $\delta \varphi_S^+(a)$  dependence on the injection level is nonlinear at negative SP. Taking into account equations (5), (12), (14) and (18), we obtain at the condition  $\varphi^S < 0$

$$\delta n_R^+ = \frac{\lambda \sigma_p j_0 \tanh(a/\lambda)}{e D \sigma [1 + S_{\text{eff}} \tanh(a/\lambda)]}, \quad (21)$$

where  $S_{\text{eff}} = (v_p \tau / \lambda) [1 + p_0 \exp(-2e\varphi^S/kT) / n_0]$  is the normalized effective SRR.

The voltage drop  $U$  across the semiconductor sample is equal to

$$U = \delta \varphi_M(-a) - \delta \varphi_M(a) = -2\delta \varphi(a). \quad (22)$$

Finally we obtain from equations (10), (11) and (19), the voltage-current relation (VCR) of the sample:

$$U = \Delta \varphi_C + \Delta \varphi_S + \Delta \varphi_R, \quad (23)$$

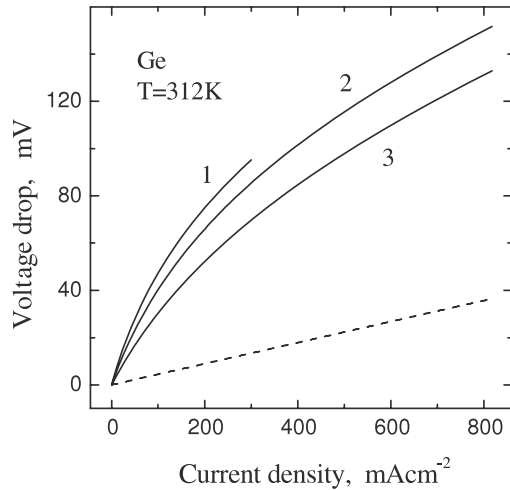
where

$$\begin{aligned} \Delta \varphi_C &= 2a j_0 \sigma^{-1}, \\ \Delta \varphi_S &= \frac{2kT}{e} \ln \left[ 1 + \frac{\delta n_R^+}{n_0} \exp\left(-\frac{e\varphi^S}{kT}\right) \right], \\ \Delta \varphi_R &= -2kT \sigma^{-1} (\mu_n - \mu_p) \delta n_R^+. \end{aligned}$$

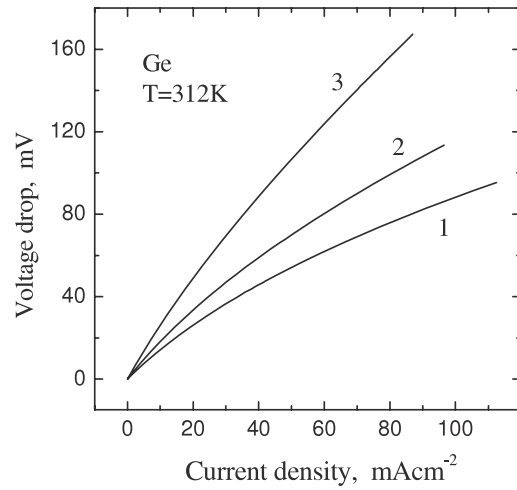
### 3. Discussion of results

It follows from equation (23) that the voltage drop  $U$  consists of several terms: the classic voltage drop across the bulk resistance of the sample  $\Delta \varphi_C$  and the emf  $\Delta \varphi_S$  ( $\Delta \varphi_R$ ) created by the S (DR) mode charge distribution. The formation of the emf  $\Delta \varphi_R$  ( $\Delta \varphi_S$ ) occurs in the following way. The flux  $j_0 e^{-1}$  transfers the non-equilibrium carriers from the surface  $x = -a$  to the surface  $x = a$ . The value of the DR mode charge density is negative in the region  $0 < x \leq a$  because the electron mobility exceeds the mobility of holes (see equation (14)). Therefore the electric potential of the DR mode  $\delta \varphi_R$  is positive in this region, i.e. it reduces the voltage drop. Since the non-equilibrium electrons cross the MSJ and the non-equilibrium holes accumulate near the surface  $x = a$ , then the value of the S mode charge density is positive near this surface. Therefore, the electric potential of the S mode  $\delta \varphi_S(a)$  is negative (see equation (19)), i.e. it increases the voltage drop. On account of the solution property  $\delta \varphi(-x) = -\delta \varphi(x)$  the non-equilibrium potential works in the same direction in the region  $-a \leq x < 0$ , i.e. the DR mode decreases the voltage drop value and the S mode increases it. Thus the barrier emf  $\Delta \varphi_S$  arises in the same direction as the current  $j_0$  flows. The emf  $\Delta \varphi_R$  works in the opposite direction (down in voltage). As follows from equations (14) and (20) the redistribution of carriers does not change the sample resistance because the relations  $\delta n(-x) = -\delta n(x)$  and  $\delta p(-x) = -\delta p(x)$  are valid. In the case of flat energy bands ( $\varphi^S = 0$ ) equation (23) coincides with that of [1]. However, the deviation from the classical Ohm's law is caused by the non-equilibrium charge, not by the redistribution of carriers.

In the case of a large SRR value ( $v_p \gg \lambda/\tau$ ) the VCR is linear because only equilibrium carriers exist in the bulk of the sample. In the case of a massive sample ( $a \gg \lambda$ ) the voltage drop  $U$  is caused in the main by the voltage drop across the bulk resistance of the sample  $\Delta \varphi_C$  and the VCR is linear too. Therefore, consider the case of a not massive sample ( $a \leq 0.5\lambda$ ) the SRR value of which is small



**Figure 1.** The VCR of the sample of Ge for some SP values: 1 –  $\varphi^S = -100$  mV, 2 –  $\varphi^S = -120$  mV and 3 –  $\varphi^S = -140$  mV. The dashed line gives the  $\Delta\varphi_c$  value.



**Figure 2.** The VCR of Ge for some sample thickness: 1 –  $a = 0.01$  cm, 2 –  $a = 0.02$  cm, 3 –  $a = 0.05$  cm.

( $v_p \leq 0.1\lambda/\tau$ ). The analysis of equations (21) and (23) shows that the voltage drop  $U$  has a maximum at the SP value  $\varphi_m^S \cong (kT/2e) \ln[v_p \tau p_0 \tanh(a/\lambda)/n_0\lambda]$ .

The VCR of a thin sample of pure Ge ( $T = 312$  K,  $\lambda = 0.1$  cm,  $a = 0.002$  cm,  $\mu_n = 3800$  cm<sup>2</sup> V<sup>-1</sup> s<sup>-1</sup>,  $\mu_p = 1800$  cm<sup>2</sup> V<sup>-1</sup> s<sup>-1</sup>,  $v_p = 20$  cm s<sup>-1</sup>) for some SP values is shown in figure 1. Curves 1 and 2 are calculated up to the current density, at which the inequality  $\delta n_R^+ \leq 0.09n_0$  is fulfilled. It is seen from figure 1 that the voltage drop across the sample for  $\varphi^S \cong \varphi_m^S$  significantly exceeds the classic one as the value obtained in [1]. Let us determine the nonlinearity coefficient (NC)  $\beta_N$  of the VCR by relation  $\beta_N(j_0) = (U/R(0)j_0) - 1$ , where  $R(j_0) = dU/dj_0$ . The NC at the current density  $j_0 = 300$  mA cm<sup>-2</sup> is equal to  $\beta_N = -0.55$  ( $\varphi^S = -100$  mV),  $\beta_N = -0.48$  ( $\varphi^S = -120$  mV) and  $\beta_N = -0.38$  ( $\varphi^S = -140$  mV). The NC has a maximum near the SP value  $\varphi_m^S$ . In the case considered here we have  $\varphi_m^S = -100$  mV.

The VCR of pure Ge ( $\varphi^S = -90$  mV) for some sample thickness is shown in figure 2. Curves 1–3 are calculated up to the current density, at which the inequality  $\delta n_R^+ \leq 0.1n_0$  is fulfilled. The NC at maximum possible current density is equal to  $\beta_N = -0.45$  (curve 1,  $j_0 = 112$  mA cm<sup>-2</sup>),  $\beta_N = -0.41$  (curve 2,  $j_0 = 96$  mA cm<sup>-2</sup>) and  $\beta_N = -0.32$  (curve 3,  $j_0 = 87$  mA cm<sup>-2</sup>).

The effect of both the voltage drop and the NC dependence on the SP strongly manifests itself in thin samples at small SRR values in the range of the SP ( $\varphi_m^S - 1.5kT/e$ ) <  $\varphi^S$  < ( $\varphi_m^S + 1.5kT/e$ ). Complex measurements of the VCR deviation from Ohm’s law for the thin samples and the photo-induced emf [3, 7] allow determination of the SRR in the real MSJ and the semiconductor SP values. It is clear that the SP determined has a maximum accuracy for small SRR values  $v_p \leq 0.1\lambda/\tau$  in the range of the SP ( $\varphi_m^S - 1.5kT/e$ ) <  $\varphi^S$  < ( $\varphi_m^S + 1.5kT/e$ ). The accuracy of the determination of the SRR and the SP values decreases outside these ranges.

Note that the surface potential depends largely on the surface levels, which form the surface energy bands in a

3D crystal [12]. One can change the surface potential by variation of the surface level density (before the formation of the MSJ).

#### 4. Conclusions

A theory of formation of the voltage across a bipolar semiconductor sample due to the current flow accounting for the energy band bending near the semiconductor surfaces has been developed. It is shown that the VCR is nonlinear at weak injection for thin samples and small enough surface recombination rates in a certain range of the surface potential. The space charge layers localized near the sample surfaces cause the nonlinearity of the VCR. The measurements of the VCR for the thin samples and the photo-induced emf allow determination of the surface recombination rate in the real metal–semiconductor junction and the semiconductor surface potential. The approach developed may be successfully used for theoretical study of any electromotive force in semiconductor samples.

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